



# **NAVAL POSTGRADUATE SCHOOL**

**MONTEREY, CALIFORNIA**

## **THESIS**

**NAVY OPERATIONAL PLANNER – UNDERSEA  
WARFARE MODULE**

by

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September 2016

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**NAVY OPERATIONAL PLANNER – UNDERSEA WARFARE MODULE**

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## **ABSTRACT**

Joint maritime operational planning is the difficult task of assigning various platforms to accomplish a multitude of missions in several areas of operations. The task becomes more difficult as resources are limited, mission requirements evolve, and platform capabilities vary. Emerging threats and technology in the undersea domain have created renewed interest and increased the priority of undersea warfare (USW) planning.

This thesis develops and provides a proof-of-concept for a decision-support tool to aid operational planning in a USW environment. Specifically, it provides an optimization model with an optimal solution that maximizes multi-mission achievement in a theater USW environment through the scheduling of surface, sub-surface, and air assets over a non-fixed time horizon.

Tactics and their mathematical representation are an input to our model. This makes the model easily adapted to any USW scenario and other warfare areas where mission achievement can be measured quantitatively.

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## **LIST OF ACRONYMS AND ABBREVIATIONS**

ASW	Anti-Submarine Warfare
CDP	Cumulative Detection Probability
CTP	Common Tactical Picture
CZ	Convergence Zone
DP	Direct Path
HVU	High Value Unit
MCM	Mine Sweeper
MIW	Mine Warfare
NMP	Navy Mission Planner
NOP	Navy Operational Planner
NOP–USW	Naval Operational Planner–Undersea Warfare
NPP	Navy Planning Process
Pd	Probability of Detection
USW	Undersea Warfare
USW–DSS	AN/UYQ-100 Undersea Warfare Decision Support System
UUV	Unmanned Undersea Vehicle

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## **EXECUTIVE SUMMARY**

Joint maritime operational planning is the difficult task of assigning various platforms to accomplish a multitude of missions in several areas of operations. The task becomes more challenging as resources are limited, mission requirements evolve, and platform capabilities vary. Technology advancement in unmanned underwater vehicles (UUV) further complicates planning efforts as the full tactical implications still have yet to be realized. Despite the complications associated with planning, emerging threats in the undersea domain have created renewed interest and increased the priority of undersea warfare (USW) planning.

The Navy planning process often uses whiteboards, simple spreadsheets, and butcher-block paper to determine the ends, ways, and means of an operation and develop courses of action. This manual planning is prone to error, lengthy in process, and does not lend itself to trade-off analysis.

To address the limitations of the current planning process, the Operations Department at the Naval Postgraduate School has continued research into scientific and mathematical-based decision-support tools with the Navy Mission Planner (NMP) and the Navy Operational Planner (NOP). The NMP is a multi-ship, multi-mission assignment planning aid that produces near optimal employment schedules on a fixed-time horizon. However, because operational planning does not occur on a fixed-time horizon, the first iteration of the NOP was developed. NOP addresses the issue of optimizing mission assignments without a fixed horizon. It introduced the concept of levels of effort, a way to track progress toward the completion of a mission through the application of ship-time. Because NOP was presented as a Mine Warfare module, it did not address platforms with varying capabilities or logistic constraints.

This research applies the NMP concept of missions located in different geographical areas and the NOP concept of a non-fixed time horizon to develop an optimization-based decision aid to support maritime operational planning in a

USW environment. Rather than applying ship-time to a mission, NOP-USW chooses the best combination of ships to apply over a small period of time. This thesis successfully establishes proof-of-concept with an integer linear programming formulation that provides a structure to tie current tactical models to campaign planning and accounts for mission requirements and logistics.

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# **I. INTRODUCTION**

## **A. NAVY OPERATIONAL PLANNING**

Joint maritime operational planning is the difficult task of assigning various platforms to accomplish a multitude of missions in several areas of operations. The task becomes more challenging as resources are limited, mission requirements evolve, and platform capabilities vary. Technological advancement in unmanned underwater vehicles (UUV) further complicates planning efforts as the full tactical implications still have yet to be realized. Despite the complications associated with planning, emerging threats in the undersea domain have created renewed interest and has increased the priority of undersea warfare (USW) planning.

### **1. Maritime Planning**

As discussed in Deleon (2015), maritime planning for contested environments is conducted at Maritime Operations Centers where commanders rely on their staff's expertise and proficiency to provide a level of planning and execution across a wide range of military operations. The tactical employment of assets has evolved as the technology on multi-mission platforms has advanced. To aid staff in determining the best tactical employment, there are now several computer-based planning aids. In anti-submarine warfare (ASW), the Undersea Warfare-Decision Support System (USW-DSS) is the primary tool used to help real time decision-making (McInvale 2016).

### **2. Navy Planning Process**

As described in Deleon (2015), the Navy Planning Process (NPP) is a six-step progression that is conducted continuously to help commanders process a multitude of information, create a coherent plan, and reevaluate as conditions change. NPP often uses whiteboards, simple spreadsheets, and butcher-block paper to determine the ends, ways, and means of an operation and develop

courses of action. This manual planning is prone to error, lengthy in process, and does not lend itself to trade-off analysis. As the scope of the operations become larger, with more assets, missions, and threats to manage, the increased difficulty of the planning task can cause commanders to rely heavily on reactionary planning and subsequently make long-range planning a low priority (Deleon 2015). A tool that connects operational objectives to local area tactics can provide a tremendous amount of information and relief from burden while a commander and his staff plan maritime operations.

## **B. LITERATURE REVIEW**

To address the limitations of the current planning process, the Navy has continued research into scientific and mathematical-based decision-support tools. Currently implemented, but still being developed, in select Navy platforms is USW–DSS, a tactical decision aid. In the Department of Operations Research at the Naval Postgraduate School, the research into decision-support tools has led faculty and students to the development of the Navy Mission Planner (NMP) (Dugan 2007) and the Navy Operational Planner (NOP) (Deleon 2015)

### **1. Undersea Warfare Decision Support System**

USW–DSS is the only tool currently available that aids the ASW commander in planning, coordinating, establishing and maintaining a common tactical picture (CTP) and in executing tactical control (Department of Navy, Naval Sea Systems Command 2016). USW–DSS develops plans for tactical engagements by using current environmental information and sensor capabilities to create asset path geometry to achieve the highest level of cumulative detection probability (CDP) against specific adversaries in a fixed region. Although it is an excellent tool for planning tactics it does not aid the commander in deciding which assets to make available when there are multiple missions competing for assets.

## **2. Navy Mission Planner**

The NMP is a multi-ship, multi-mission assignment planning aid that produces near-optimal employment schedules on a fixed time horizon. The model relies on a predefined set of mission requirements, ship capabilities, and an enormous list of possible schedules. NMP provides the initial concept of multi-missions associated to geographical areas (Deleon 2015). This initial model was cumbersome but through follow-on research by Silva (2009) and Hallman (2009), the computational burden was reduced, logistic planning capabilities were included, and was proven effective during the planning of Trident Warrior 2009 (Deleon 2015).

## **3. Navy Operational Planner**

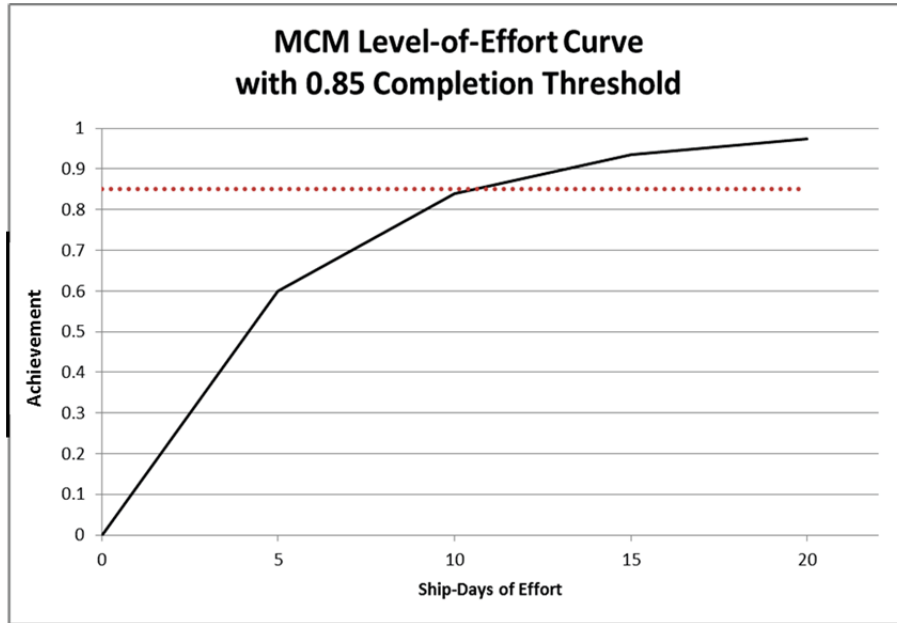
The purpose of the NOP is to take the concepts from NMP, a logistics model, and apply them to operational planning, which does not occur on a fixed time horizon. The first iteration of NOP is presented as a Mine Warfare module and addresses the issue of optimizing mission assignments without a fixed horizon. It is designed to advise theater commanders on how to allocate multiple ships to multiple missions in order to complete those missions to a prescribed level and reduce the overall time for forces to advance to the next phase of the campaign. The research introduces the concept of *levels of effort*, a way to track progress toward the completion of a mission through the application of ship-time.

Although the underlying purpose of the NOP is to provide a model that can be tailored to other warfare areas, there are two issues that prevent adaptation. The first is the assumption that the application of one ship-day always results in the same amount of progress, or *achievement*, for a mission. When the capabilities of the platforms can vary this assumption is no longer true. The second issue is that the NOP formulation did not account for several logistic considerations that are part of planning, such as the distance between missions and the at-sea (or on-station) endurance of the platforms.

#### 4. Issues with Adapting NOP to Other Mission Areas

The NOP uses the concept of applying ship-time in its formulation. Illustrated in Figure 1 is a piecewise linear form of the equation  $Achievement = 1 - e^{-\gamma t}$ , where *Achievement* is the probability that a mine field has been cleared,  $\gamma$  is the rate of clearance for a single Mine Sweeper (MCM), and  $t$  is the amount of time a ship has been sweeping. The NOP assumption is that all ships have the same  $\gamma$  and therefore have the same curve.

Figure 1. Example NOP Linearized Level-of-Effort Curve



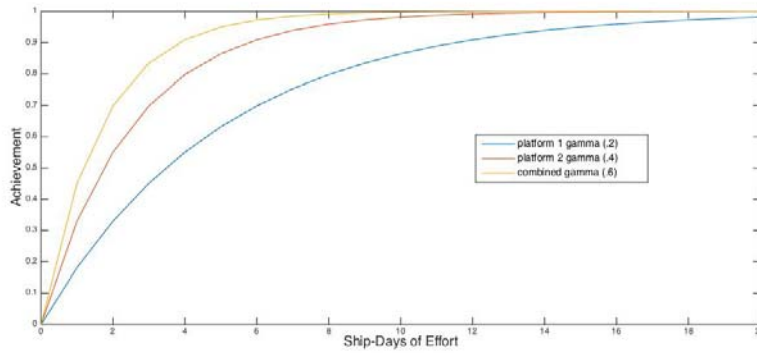
An example of the application of ship-time: Accomplishing the mission from 0.0 to 0.6 takes five ship-days of effort. This can be accomplished with one ship over five days or five ships over one day. Both options will give the same result.

However, anti-submarine warfare is not done by a single platform type; instead, different platforms have different values of  $\gamma$  for the same mission. Using a random search as a conservative example, the equation appears the same as the mine-clearing model, however  $\gamma$  has a slightly different meaning as a search-effectiveness rate equal to  $\frac{\text{Sensor Sweep Width} * \text{Searcher Velocity}}{\text{Search Area}}$ .



Therefore, as capabilities vary between assets, so does the corresponding  $\gamma$ . If a set of  $n$  assets with parameters  $\gamma_1, \gamma_2, \dots, \gamma_n$  are simultaneously searching, the resulting equation is  $Achievement = 1 - e^{-(\gamma_1 + \gamma_2 + \dots + \gamma_n) * t}$ , where  $t$  is now the total ship-time spent searching (see Figure 2).

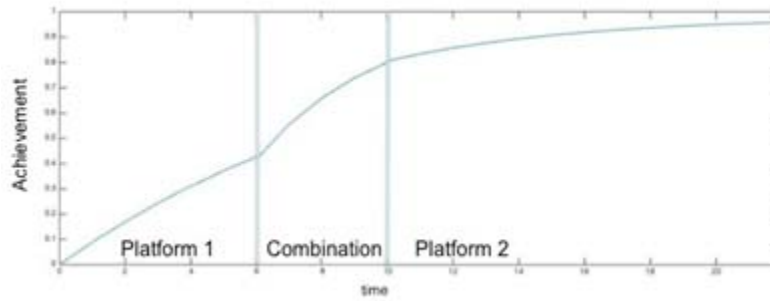
Figure 2. Level-of-Effort Curve with Varying Platform Capabilities



This graph is derived using the same achievement equation from NOP but with three different gamma values. Platform 1 has a value of .2, platform 2 has a value of .4 with a combined platform value of .6.

Figure 2 illustrates how the curves vary, and therefore how the simple concept of ship-time cannot be directly applied in NOP-USW; interchanging of number of platforms and ship-days applied is not possible. An achievement curve in NOP-USW may look more like Figure 3, where the curve is not smooth and, in particular, where the rate of increase depends on the combination of searching platforms that are active in any given time period.

Figure 3. Single Mission NOP-USW



This graph shows how a single mission could progress with two platforms that have different  $\gamma$ . Platform 1 searches until time 6 when it is joined by platform 2. At time 10, platform 1 departs and platform 2 completes the search.

## II. NAVY OPERATIONAL PLANNER–UNDERSEA WARFARE

### A. DESCRIPTION

This research applies the NMP concept of missions located in different geographical areas and the NOP concept of a non-fixed time horizon to develop an optimization-based decision aid to support maritime operational planning in a USW environment. This type of decision aid will give theater commanders more confidence with the effort it may take to have sea control and the timing it will take to transition between phases of operations.

#### 1. Reconceptualizing the Problem

Because we need to account for heterogeneous assets that can cooperate in USW missions, our formulation for NOP–USW is significantly different from previous research. Here we present the primary modeling features of NOP–USW in order to motivate our mathematical programming formulation.

##### a. *Discrete Time Steps*

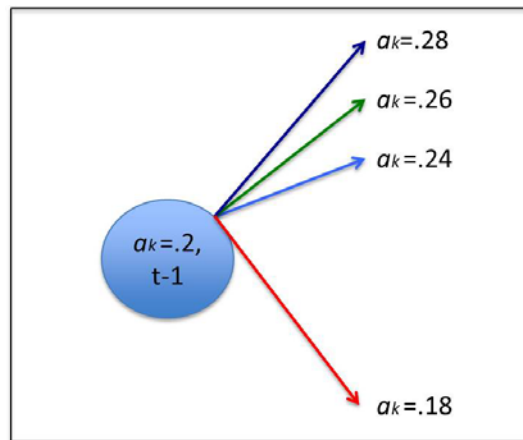
NOP–USW suggests operational plans over a set of discrete time steps. Depending on the design of the scenario, such as the size of the missions and their geographic distances, and various platform characteristics the resolution for time could be one hour to one day. The length of each time step that is required for each scenario is a result of the operational tempo of the assets in the scenario. Fast moving assets such as aircraft will need a higher resolution where the opposite is required for a slower moving asset such as a submarine.

Rather than applying total ship-time to a mission, NOP–USW chooses a *combination* of platforms to apply over each time period, and calculates the increase in achievement that results from that particular combination of assets. Different types of platforms can work together in combinations to achieve faster clearing rates than if they were independent. This choice is based on progress already achieved and the relative rate of return of the different combinations.

### **b. Achievement**

The level of achievement of a mission is *not* defined by a continuous function of time but via a set of discrete *achievement values*. Reaching a particular achievement value depends on the prior value reached in the previous time step and the combination applied. In some cases no assets are applied, we represent this with an “empty” combination which can eventually lead to a decay in achievement. The set of achievement levels is indexed in our models by  $k$ , and the actual achievement level associated with each  $k$  is  $a_k$  (see Figure 4).

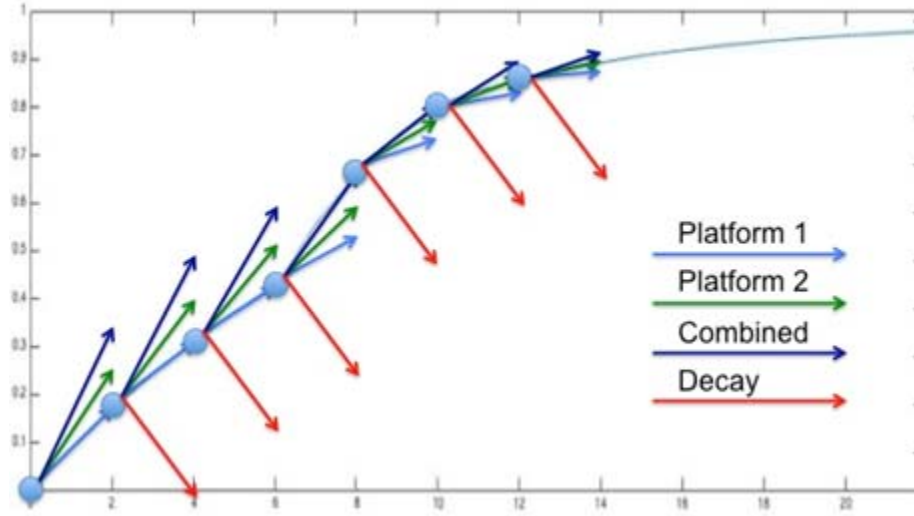
Figure 4. Single Transition of Achievement



This figure shows the discrete levels of achievement available from a single level of achievement from a previous time step. In this case there are three combinations of platforms that will improve the level of achievement and one case where a mission decays.

When these single transitions are applied over many time steps, the increases in achievement levels begin to look like recognizable mission progress (see Figure 5).

Figure 5. Single Mission Achievement

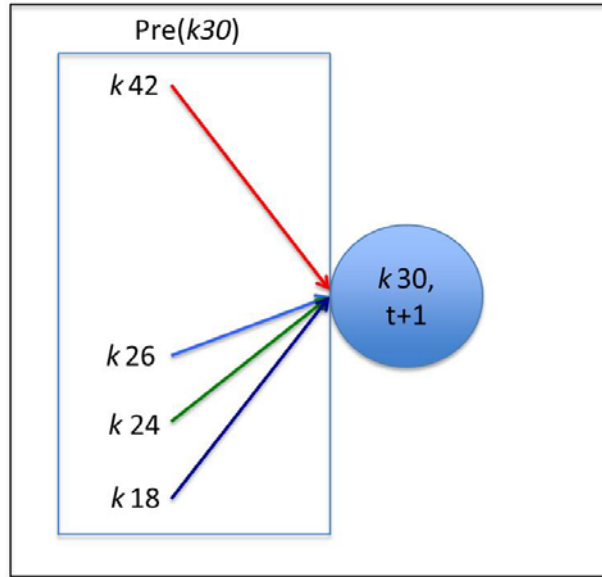


The curve shows a choice of combinations over 14 time steps. In the first 6 steps, platform 1 is achieving the mission to the .4 level by itself. At  $t=6$ , platform 2 joins the mission and combined gammas are used for an additional 4 time steps taking the achievement up to .8. At  $t=10$ , platform 1 departs and platform 2 makes the final jumps in achievement.

## 2. Predecessors of $k$

In order to determine whether a particular plan has achieved a value  $a_k$  by time  $t$ , we calculated the set of all possible levels of achievement from which level  $k$  could be reached and, for each of those levels, the corresponding combination that provides the transition to level  $k$ . We refer to these levels as the *predecessors* of  $k$  (see Figure 6).  $pre(k)$  is the set of all combinations of platforms that can lead to  $k$  from different preceding levels of achievement.

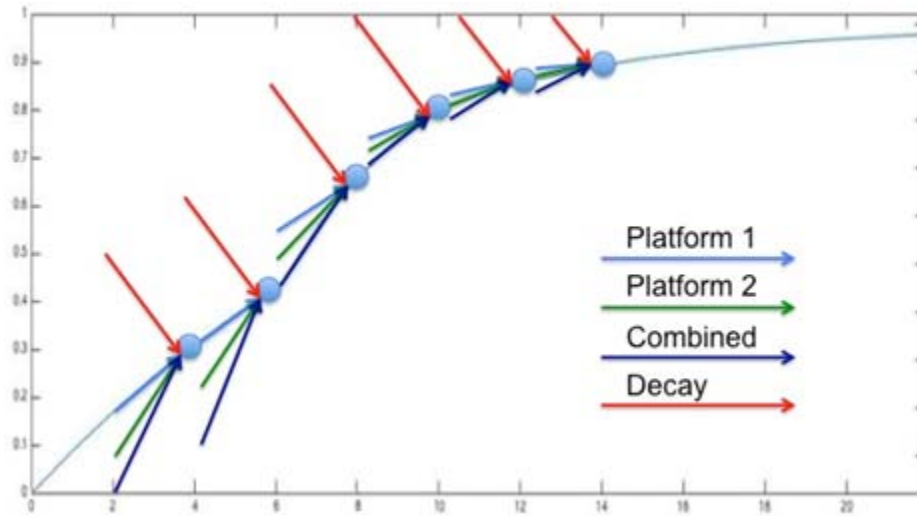
Figure 6. Single Transition of  $pre(k)$



This figure shows the discrete levels of achievement that lead to a single level of achievement in the next time step. In this case there are three combinations of platforms that work to achieve  $k30$  and one case where mission decay (because no assets are assigned to that mission) leads to  $k30$ .

The development of  $pre(k)$  is based on tactical models. The increase or decrease from one  $a_k$  to another by all platforms and combinations is determined by fitting analytical models to the achievement scale (see Figure 7). In USW these models generally fall into three categories: area search, barrier search, and mine clearance. Because of the exponential nature of these models the spacing of  $k$  variables from zero to one is logarithmic to give higher resolution at the upper values where it generally takes more effort in time for a platform to achieve smaller increases.

Figure 7. Developing  $pre(k)$



Using the same scenario as Figure 5, Figure 7 shows achievement rates as a result of choosing the best available combination defined by  $pre(k)$ .

### 3. New Concepts

Three concepts are considered in NOP-USW that are not in previous research: time phasing of missions, mutually exclusive missions, and asset availability. Time phasing of missions allows a commander to plan in longer time horizons. In ASW, the uncertainty of a target's position grows as time increases without detections. This uncertainty growth is modeled in several ways but using that information a commander could choose to have missions become available and then unavailable at different time steps. Mutually exclusive missions give flexibility so that missions that occur in the same location but are counter-productive are not assigned to assets at the same time. Asset availability helps a theater commander in that all of his assets may not be available on day one of the operation and may still be transiting from another theater.

#### B. MULTI-MISSION ACHIEVEMENT MODEL

##### 1. Sets and Indices [Cardinality]

$t \in T$  Time periods in planning horizon [ $\sim 36$ ]

$m \in M$  Set of missions [ $\sim 4$ ]

$k \in K$	Discrete levels of achievement [ $\sim 100$ ]
$p \in P$	Set of specific platforms [ $\sim 3$ ]
$c \in C$	Set of combinations of platforms [ $\sim 7$ ]
$p \in CP_c$	Platform $p$ is in combination $c$
$m \in TM_t$	Mission $m$ exists in period $t$
$pre(k)$	Set of $(k',c)$ conditions that precede achievement level $k$ . $pre(k30)=[k5,(ship,sub)],[k15,(ship,helo)],[k20,(ship)],\dots\}$
$p \in TP_t$	Platform $p$ available in period $t$

## 2. Derived Set

$c \in TC_t$	Combination $c$ available in period $t$
$(t,c) \in TC \Leftrightarrow (t,p) \in TP \forall p \in c$	

## 3. Data [Units]

$a_k$	Numerical value of achievement level $k$ [0.0-1.0]
$value_m$	Priority value of mission $m$ [1-5]
$thresh_m$	Threshold fraction required for accomplishing mission $m$ [0.0-1.0]
$d_{m,m',p}$	Travel time required between missions $(m,m')$ per platform $p$ [number of time periods $t$ ]
$ose_p$	On-station endurance of platform $p$ [number of time periods $t$ ]
$ase_p$	At-sea endurance of platform $p$ to be at sea or in the air [number of time periods $t$ ]



$dt_p$  Amount of downtime required in port for platform  $p$  once  $ase_p$  is exceeded [number of time periods  $t$ ]

#### 4. Variables [Units]

$KACH_{t,m,k}$  Achievement level  $k$  is feasible at time  $t$  for mission  $m$  [Binary]

$CACT_{t,m,c}$  Combination of platforms  $c$  is chosen for mission  $m$  in  $t$  [Binary]

$DONE_{t,m}$  Mission  $m$  achievement meets or exceeds its threshold in time  $t$  [Binary]

$MACT_{t,m}$  Mission  $m$  has assets assigned at time  $t$  [Binary]

$ASGND_{t,m,p}$  Platform  $p$  is assigned to mission  $m$  at time  $t$  [Binary]

$KCACT_{k,c,t,m}$  Mission  $m$  achievement is at or above level  $k$  in  $t$  and combination  $c$  is applied to mission  $m$  in  $t$  [Binary]

$EMPLYD_{t,p}$  Platform  $p$  is employed in time  $t$  [Binary]

$ATSEA_{t,p}$  Platform  $p$  is at sea in time  $t$  [Binary]

$PDOWN_{t,p}$  Platform  $p$  is in port or refueling in time  $t$  [Binary]

## 5. Formulation NOP-USW

$$\begin{aligned}
\max \quad & \sum_{t,m} (DONE_{t,m} + (0.01)MACT_{t,m}) + \sum_{t,m,k} (0.1)value_m a_k KACH_{t,m,k} \quad (M0) \\
s.t. \quad & KCACT_{t,k,m,c} \leq KACH_{t,m,k} \quad \forall k, t, m \in TM_t, c \in TC_t \quad (M1) \\
& KACH_{t,m,k} \leq \sum_{(k',c):(k',c,m) \in PRE_k} KCACT_{t-1,m,k',c} \quad \forall k, t > 1, m \in TM_t \quad (M2) \\
& KCACT_{t,c,m,k} \leq CACT_{t,m,c} \quad \forall k, t, m \in TM_t, c \in TC_t \quad (M3) \\
& CACT_{t,m,c} \leq ASGND_{t,m,p} \quad \forall t, m \in TM_t, c \in TC_t, p \in TP_t \quad (M4) \\
& \sum_{c \in TC_t} CACT_{t,m,c} \leq 1 \quad \forall t, m \in TM_t \quad (M5) \\
& MACT_{t,m} \leq \sum_{p \in TP_t} ASGND_{t,m,p} \quad \forall t, m \in TM_t \quad (M6) \\
& ASGND_{t,m,p} \leq EMPLYD_{t,p} \quad \forall t, m \in TM_t, p \in TP_t \quad (M7) \\
& \sum_m ASGND_{t,m,p} \leq 1 \quad \forall t, p \in TP_t \quad (M8) \\
& ASGND_{t,m,p} \leq \sum_{c \in TC_t} CACT_{t,m,c} \quad \forall t, m \in TM_t, p \in TP_t \quad (M9) \\
& \sum_{(k',c):(k',c,m) \in pre(k)} KCACT_{t,m,k',c} \leq 1 \quad \forall k, t, m \in TM_t \quad (M10) \\
& DONE_{t,m} \leq \sum_{k: d_k \geq thresh_m} KACH_{t,m,k} \quad \forall t, m \in TM_t \quad (M11) \\
& ASGND_{t,m,p} + ASGND_{t',m',p} \leq 1 \quad \forall p, t, t', m, m': p \in TP_t \cap TP_{t'}, \quad (M12) \\
& \quad m \in TM_t, m' \in TM_{t'}, (t'-t) < d_{m,m',p} \\
& \sum_{t \leq t' \leq ose_p} EMPLYD_{t',p'} \leq ose_p \quad \forall t, p \in TP_t, t \leq ose_p \quad (M13) \\
& \sum_{t \leq t' \leq ase_p} ATSEA_{t',p'} + pt_p \leq ase_p \quad \forall t, p \in TP_t, t \leq ase_p \quad (M14) \\
& EMPLYD_{t,p} \leq ATSEA_{t,p} \quad \forall t, p \in TP_t \quad (M15) \\
& \sum_{t \leq t' \leq t+dt_p} PDOWN_{t',p} \geq dt_p \quad \forall t, p \in TP_t, t \leq T - dt_p \quad (M16) \\
& PDOWN_{t,p} + PDOWN_{t+dt_p,p} \leq 1 \quad \forall t, p \in TP_t, t \leq T - dt_p \quad (M17) \\
& ATSEA_{t,p} + PDOWN_{t,p} = 1 \quad \forall t, p \in TP_t \quad (M18)
\end{aligned}$$

$$\begin{array}{lll}
DONE_{t,m} & \in \{0,1\} & \forall t,m \in TM_t \\
MACT_{t,m} & \in \{0,1\} & \forall t,m \in TM_t \\
KCACT_{t,k,m,c} & \in \{0,1\} & \forall t,m \in TM_t, k,c \in TC_t \\
KACH_{t,m,k} & \in \{0,1\} & \forall t,m \in TM_t, k \\
CACT_{t,m,c} & \in \{0,1\} & \forall t,m \in TM_t, c \in TC_t \\
ASGND_{t,m,p} & \in \{0,1\} & \forall t,m \in TM_t, p \in TP_t \\
EMPLYD_{t,p} & \in \{0,1\} & \forall t,p \in TP_t \\
ATSEA_{t,p} & \in \{0,1\} & \forall t,p \in TP_t \\
PDOWN_{t,p} & \in \{0,1\} & \forall t,p \in TP_t
\end{array}$$

## 6. Discussion

### a. Objective

The objective, equation (M0), calculates the level of achievement of missions achieved with a bonus for completing missions to their threshold and a penalty for not actively working missions.

### b. Achievement Constraints

Equation (M1) forces the level of achievement  $k$  to 1 if a  $(k,c)$  combination is activated and equation (M10) only allows one of  $k$ 's predecessors to be activated. From equation (M2), a level of achievement can only be activated if a combination in its  $pre(k)$  is activated. equation (M3) only allows a combination to be activated if a corresponding  $k$  has been achieved. In equation (M4) if a combination is activated for a mission all of the associated platforms are activated and in equation (M5) only one combination can be activated per mission. Equation (M6) will only allow a mission to be active if a platform is assigned. From equation (M7), if a platform is assigned to at least one mission that platform is considered employed in the time period. Equation (M8) ensures a platform is only assigned once per mission. Equation (M9) only allows a platform

to be assigned if a combination containing it has been activated. Equation (M11) tracks the completion of missions.

***c. Logistic Constraints***

Equation (M12) prevents platforms from being considered assigned if they are transiting between missions. Equation (M13) will not allow platforms to perform a mission longer than their on station endurance. An example of a factor that would limit endurance is sonobuoy endurance deployed from an aircraft. Equation (M14) prevents a platform from being at sea longer than its at-sea endurance. This constraint assumes there is an average amount of time that a platform takes to go from mission areas to the nearest port. Equation (M15) ensures that only a platform at sea can be considered employed. Equation (M16) and equation (M17) dictate that if a platform is in port it must be in port for a set amount of time and those time steps must be continuous. Equation (M18) ensures that the platform is either at sea or in port.

### III. RESULTS

#### A. GENERAL INFORMATION FOR ALL SCENARIOS

The scenario we used for testing our model is a region with four missions where area search is required against a generic enemy submarine. The assets available are a submarine, referred to as “sub,” a surface ship (“ship”), and a maritime patrol aircraft (MPA) (“p8”). The missions vary by depth in order to have variation in sweep width (W) of the platform and in surface area to illustrate different tactical inputs to the model. We assume the enemy submarine has one of two speeds, 1 kt and 15 kts, in order to give conservative tactical estimates depending on the tactical model used. Each time period represents 6 hours, and we have 36 periods for a total available time horizon of 9 days (see Tables 1–4.)

##### 1. Data

###### a. *Mission information*

We included depth and area to highlight the model’s ability to handle performance variations of a particular platform’s sensors while conducting different missions. The threshold and value are inputs made by the commander and his staff.

Table 1. Specific Mission Information

$m$	Depth	Area	$thresh_m$	$value_m$
Mission 1	250 m	4700 sq. nm	0.85	2
Mission 2	300 m	3000 sq. nm	0.83	5
Mission 3	100 m	1000 sq. nm	0.81	3
Mission 4	100 m	1200 sq. nm	0.86	4

Table 2. Distance Between Missions

	Mission 2	Mission 3	Mission 4
Mission 1	100 nm	100 nm	0 nm
Mission 2		150 nm	100 nm
Mission 3			100 nm

**b. Platform Information**

The option of having a convergence zone (CZ) or direct path (DP) sweep width is a function of the depth associated with a mission. Missions that have depth less than 200 m limit platforms to DP and greater than 200 m allow CZ.

Table 3. Waterborne Vehicles

	Ship 1	Sub 1
Speed	15 kts	12 kts
W CZ	35 nm	35 nm
W DP	1.5 nm	2 nm
At-sea endurance	336 hrs	672 hrs
Availability	12 hrs	36 hrs

The air asset for the scenario is assumed to only be able to control 24 buoys at one time for 6 hours.

Table 4. Airborne Vehicle

	Air 1
Speed	300 kts
On-station endurance	6 hrs
At-sea endurance	12 hrs
W buoy	2.5 nm
Controllable Buoys	24
Availability	24 hrs

## 2. Tactics

We used two tactical models in the scenario to develop  $pre(k)$ . The Random Search Model and the Stationary Sensor Model both provide a generic representation of how a sensor may perform in area searches. Both models are considered to provide lower bounds for the performance of any reasonable search scheme.

### a. Random Search

The Random search model is a probability model based on the on the time to increase coverage of a search area. The model relies on three inputs: sensor sweep width, size of the search area, and the speed of the searcher to determine probability of detection ( $Pd$ ). Random search has the form  $Pd = 1 - e^{-\gamma t}$  where  $\gamma = \frac{Sweep\ Width * Velocity}{Search\ Area}$ . This model is used in any case where either the ship or submarine are searching alone or together. In the case where the searchers work together their respective  $\gamma$  values are summed to create the resulting the exponent; this is a result of assuming that those platforms are searching the same area independently, and do not provide any extra (i.e., synergistic, through cueing) benefits to each other while searching.

### b. Stationary Sensor Search

The stationary sensor model is used to model the tactics of the MPA. The stationary sensor model is slightly different in that it relies on the enemy speed and accounts for the instantaneous gain in cumulative probability of detection when the sensor is initially activated in the search area. Stationary search has the form  $Pd = 1 - \left(1 - \frac{N * A_B}{A_T}\right) e^{-\alpha t}$  where  $N$  is the number of buoys,  $A_B$  is the square area of the sweep width,  $A_T$  is the search area, and  $\alpha = \frac{Sensor\ Sweep\ Width * Target\ Velocity}{Search\ Area}$ . In this scenario the MPA never searches

alone and only enhances the search of the other two platforms. In the case where the platforms are combined the model has the form

$$Pd = 1 - \left( 1 - \frac{N * A_B}{A_T} \right) e^{-(\alpha + \gamma_{sub} + \dots + \gamma_{ship})t}.$$

### **c. Other Models**

There are several other search models that could have been used in NOP. Barrier search, spiral search, and mine clearance are examples of other models that can work as inputs to NOP-USW. The limiting factor for choosing models is if they can be represented as a CDP, whether step-wise or continuous. This feature is what will allow NOP-USW to be integrated with systems like USW-DSS.

## **B. SCENARIO RESULT**

The model was implemented using Pyomo (Hart, Laird, Watson, and Woodruff, 2011) and solved using the Gurobi solver (Gurobi Optimization, Inc. 2016). All model runs were conducted using a 1.3Ghz Intel Core i5 processor with 4.0 GB of RAM, running the OSX Version 10.9.5 operating system. The main scenario model has 74,758 equations and 63,035 binary variables, all of which are binary. The model formulation for our main scenario solves within 5,501 seconds at a 293% optimality gap. This scheduling problem is difficult to solve to optimality, but after looking at the resulting solutions it seems that the model is having the most trouble reducing the upper bound; the solutions themselves seem to be reasonably close to an optimal schedule.

### **1. Platform schedule**

The resulting schedule is shown in Table 5. It appears that platforms do not change between missions unless there is little time cost between them such as with missions 1 and 4.



Table 5. Platform Schedule

	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20
Ship	m2						m2		m2	m2	m2	m2	m2	m2	m2
Sub		m2	m2	m2	m2	m2		m2		m1	m1	m1	m1	m1	m1
MPA					m2				m2						

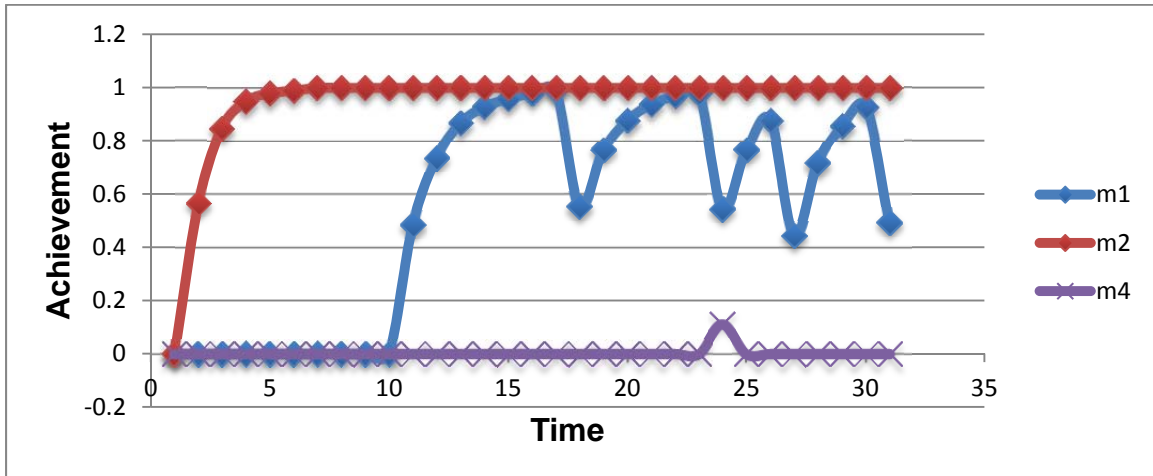
  

	t21	t22	t23	t24	t25	t26	t27	t28	t29	t30	t31	t32	t33	t34	t35
Ship	m2	m2	m2	m2	m2	m2	m2	m2	m2	m2	m2	m2	m2	m2	m2
Sub	m1		m1	m1	m1	m1	m1	m4	m1	m1		m1	m1	m1	
MPA	m2						m2			m1			m1	m1	m2

## 2. Mission Achievement

Figure 8 shows the level of achievement for each mission. The random search model, stationary sensor model, and decay are apparent in the graph. The effect of mutual exclusion between mission 1 and mission 4 is seen at time step 24. Because of the high value of mission 2 it is constantly pursued where as mission 1 has a lower value and effort only brings the achievement above the threshold. We do not have enough assets to pursue all missions simultaneously, and so we see that the solution found by the model does not put much effort at all into the lowest priority mission.

Figure 8. Mission Progress



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## **IV. CONCLUSION**

### **A. SUMMARY**

NOP-USW is a planning aid to complement manual operational planning efforts. NOP-USW suggests the correct allocation of assets across a wide theater of operation to accomplish missions in the most efficient manner. It also gives the commander a notion of the time and effort it will take to advance to the next phase of operations. This knowledge can help a commander understand the tradeoffs in the balance of time, space, and force in a maritime environment.

NOP-USW establishes a proof-of-concept of an integer linear programming model for operational planning that provides one possible architecture to link current tactical models to campaign planning, and accounts for both mission requirements and logistics.

### **B. APPLICABLE SCENARIO EXAMPLES**

The scenarios that can be evaluated by this model are only limited by creativity. The example tested in this thesis was a simple search scenario. However, the model is designed to handle far more complicated scenarios. It allows time phasing of the missions, which accommodates the strategic continuum of ASW to stop the enemy in port, defeat them at choke points, defeat them in the open ocean, and then defend allied assets. Time phasing of missions also allows for the planning of clearing a minefield before campaigns begin.

In homeland defense responding to a MIW threat can be incredibly difficult to plan when the number of ports threatened becomes more than, say, two. However, the CDP for clearing harbors, the acceptable level of risk, and travel times between ports are well known or are being actively studied and would fit easily into NOP-USW.

Unmanned undersea vehicles are developing at a remarkable rate. With their development comes the question of how to have command and control and

how to plan their effective use. Because the mission algorithms and parameters for UUVs are preprogrammed, NOP-USW can be used immediately to plan the employment of hundreds of them.

## **C. FOLLOW ON WORK**

### **1. In-Depth Analysis in Other Warfare Areas**

Analysis of other maritime warfare areas must be conducted for NOP to realize the complexity of maritime operational planning. Integrating the concept of levels of achievement and preceding sets, as the mechanism for achievement into models for other warfare areas is a significant next step. This will allow the creation of an NOP that includes multiple warfare areas interacting.

### **2. Model improvements**

#### ***a. Further Testing***

Varying scenarios should be run through the model to test its robustness as well as to identify any minor issues with the formulation. Variations should include changing distances between missions to ensure tradeoffs in platform logistics reflect real world decisions. Testing mutually exclusive missions will require a significant amount of pre-processing data and possible adjustment to the formulation. Two other items that should be varied in order to test the flexibility and robustness of the model are the number and type of assets, and number of missions.

#### ***b. Slow Run Times***

There are several factors that lead to slow run time. The number of variables is the largest contributor. This is in part due to the resolution of achievement levels. Future research should include finding the optimal resolution depending on the tactics used for input and other areas that can lead to faster run times. It should also include reformulating certain constraints to reduce the total number of equations. Finally, future research should explore adding cuts to reduce complexity and increasing speed.

**c.     *Weighting***

The components of the objective are weighted. Further research should explore ways of determining appropriate weights of these components to improve the usability of the model for campaign analysis. This weighting will include capturing concepts like commander's intent and whether missions involve durable achievement (i.e., once a mission has been "completed," we do not have to worry about it again) or whether any progress, even that above a threshold, is a transient achievement and needs to be maintained to the end of the model planning horizon.

**d.     *Improving the Logistic Component (Average Time in Port)***

Adding port/platform indexing and a distance component to the downtime for platforms will be an important addition to increase the fidelity of the model.

**e.     *Moving Missions (HVU)***

Adding an option to make distance between missions a function of time would allow the concept of missions surrounding the transit of a high value unit to be included. The distance data will have to be indexed by time as well as by mission pairs and platform. Also, platform speed will have to be carefully considered in deciding if it is possible for a platform to get on station and remain a contributor for any significant mission duration.

**3.     **Integrating Tactical Systems****

A logical final step is to integrate our model with existing tactical decision aids such as USW-DSS. This will take a level of effort beyond the scope of a single master's thesis, but could be handled by skilled software developers working in conjunction with the appropriate command. For integrating with USW-DSS, Undersea Warfare Development Center detachment San Diego is the appropriate contact.

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